

Second Edition

Philip Hartman

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In Applied Mathematics

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Ordinary Differential Equations

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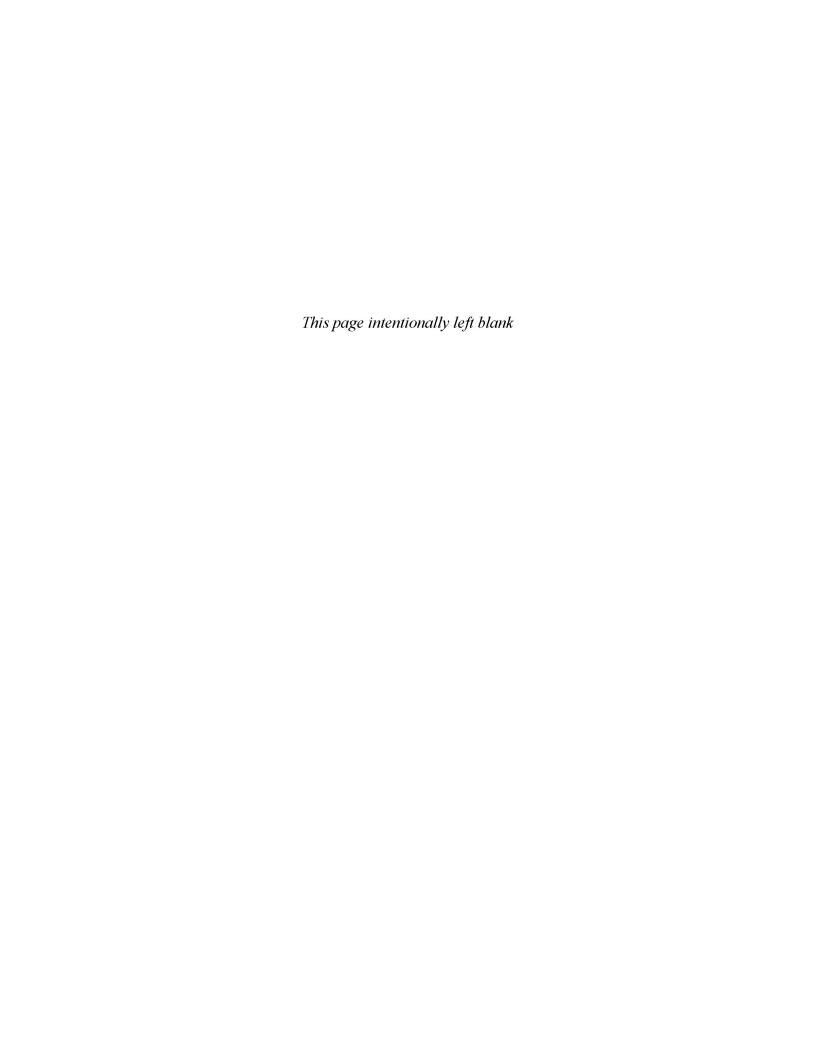
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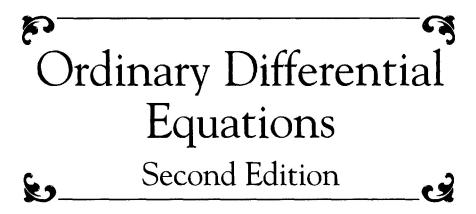
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Philip Hartman, Ordinary Differential Equations, Second Edition





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This SIAM edition is an unabridged, corrected republication of the edition published by Birkhäuser, Boston, Basel, Stuttgart, 1982. The original edition was published by John Wiley & Sons, New York, 1964.

10987654321

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Library of Congress Cataloging-in-Publication Data

Hartman, Philip, 1915-

Ordinary differential equations / Philip Hartman.

p. cm. - (Classics in applied mathematics; 38)

Previously published: 2nd ed. Boston: Birkhäuser, 1982. Originally published: Baltimore, Md., 1973.

Includes bibliographical references and index.

ISBN 0-89871-510-5 (pbk.)

1. Differential equations. I. Title. II. Series.

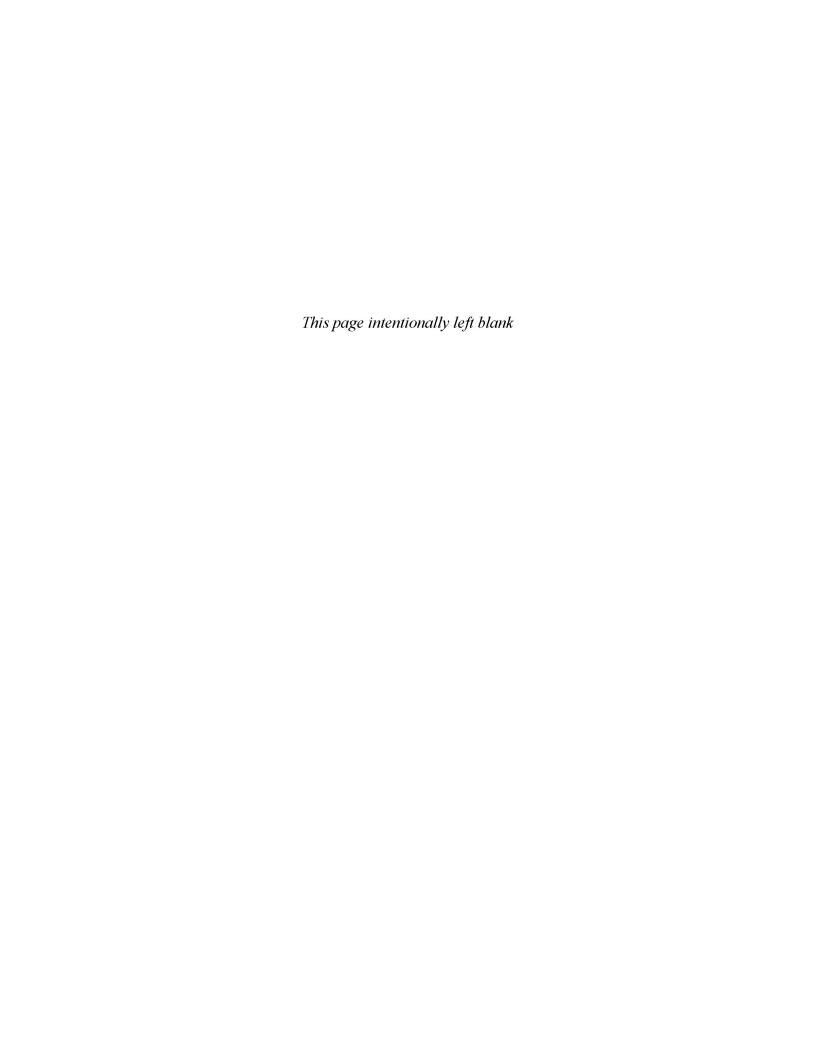
QA372 .H33 2002 515'.352-dc21

2002017641

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To the memory of my parents

To the patience of Sylvia, Judith, and Marilyn



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